(8)

6/29/2019

2h

①

Estuarine + Coastal Fluid Dynamics

from the Latin "aestus," = boiling, tide

What makes the edges of the ocean special?

- strong tidal currents ⇒ turbulent mixing
- rough, shallow bathymetry
  ⇒ form drag, hydraulic control
- rivers ⇒ strong lateral density gradients
- coastal boundary ⇒ Ekman transport
  rapidly creates pressure gradients,
  Kelvin waves, shelf waves, upwelling
- Coriolis important but often not dominant.
- Concentrated biological productivity
  and pollution.

Scales

I • most geophysical flows have thin "aspect ratio" meaning:

$$\left[\frac{\partial}{\partial x}\right] \sim \frac{1}{L} \quad \text{and} \quad \left[\frac{\partial}{\partial z}\right] \sim \frac{1}{H} \quad \text{where } [\;] \equiv \text{"scale of"}$$

then $H/L \ll 1$

Consequences: $[w] \ll [u, v]$ small vertical velocity

also: pressure ~ hydrostatic

Exceptions: fronts, surface gravity waves, internal waves with $\omega \sim N$

II • density variations are small

Density: $\rho = \rho_0 + \rho'(\underline{x}, t)$ — density variation; ρ_0: constant "background" density

$$\left.\begin{array}{l}[\rho_0] \sim 1000 \text{ kg m}^{-3} \\ [\rho'] \sim 1\text{-}25 \text{ kg m}^{-3}\end{array}\right\} \Rightarrow \frac{[\rho']}{\rho_0} \ll 1$$

Leads to simplifications in the momentum and mass conservation eqns. called the "Boussinesq approximation."

III • we average over turbulent time/space scales : "Reynolds averaging."

Full velocity: $u_f = u + u'$

u ← Reynolds averaged velocity

u' ← turbulent fluctuations

$\langle\ \rangle$ = average over turbulence

$\Rightarrow \langle u_f \rangle = u \ , \quad \langle u' \rangle = 0$

Consider the $\boxed{\text{x-mom}}$ material derivative :

Rate of change of u following a fluid parcel : $\dfrac{Du_f}{Dt} = \dfrac{\partial u_f}{\partial t} + \underline{u}_f \cdot \nabla u_f \quad (*)$

Assume incompressible $\nabla \cdot \underline{u}_f = 0$, add $u_f(\nabla \cdot \underline{u}_f)$ to $(*)$

$\Rightarrow \dfrac{Du_f}{Dt} = \dfrac{\partial u_f}{\partial t} + \nabla \cdot (u_f \underline{u}_f)$, and take average

$$\left\langle \frac{Du_f}{Dt} \right\rangle = \underset{\displaystyle \downarrow \atop \displaystyle \frac{\partial u}{\partial t}}{\left\langle \frac{\partial u_f}{\partial t} \right\rangle} + \langle u_f u_f \rangle_x + \langle u_f v_f \rangle_y + \langle u_f w_f \rangle_z$$

$$\langle u_f w_f \rangle_z : \quad (uw)_z + \underset{\displaystyle \downarrow \atop 0}{\langle u w' \rangle_z} + \underset{\displaystyle \downarrow \atop 0}{\langle u' w \rangle_z} + \langle u' w' \rangle_z$$

$-\langle u'w' \rangle$ is called the "Reynolds stress"

(really stress / ρ because "stress" means force / unit area)

stress sign convention 

Any systematic correlation of u' + w' will move u momentum around.

<u>Assume</u> it acts like Fickean diffusion:

$$-\langle u'w' \rangle = A \frac{\partial u}{\partial z}$$

A = "eddy viscosity"

$A \sim 10^{-2}$ m^2/s in a boundary layer

(<u>vastly</u> larger than ν, molecular kinematic viscosity $\sim 10^{-6}$ m^2/s)

Typically $$A \frac{\partial u}{\partial z} \gg A \frac{\partial u}{\partial x} + A \frac{\partial u}{\partial y}$$

because $\frac{H}{L} \ll 1$

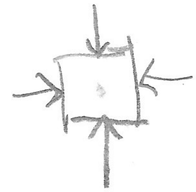
Using these three scaling & averaging assumptions, we have simplified momentum conservation equations:

Physics: Force = mass × acceleration

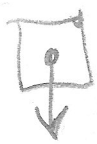
Fluid Mech.: acceleration = $\dfrac{\text{Force}}{\text{volume}} \Big/ \left(\dfrac{\text{mass}}{\text{volume}}\right) \leftarrow \rho$

or $\dfrac{Du}{Dt} = \dfrac{1}{\rho}\Big\{$ sum of $\dfrac{\text{force}}{\text{volume}}$ due to:

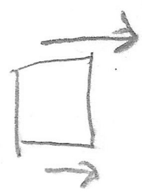
pressure



gravity



turbulent stress divergence



} and Coriolis "force"

x mom: $$\frac{Du}{Dt} - fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\left(A\frac{\partial u}{\partial z}\right)$$

y mom: $$\frac{Dv}{Dt} + fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{\partial}{\partial z}\left(A\frac{\partial v}{\partial z}\right)$$

Boussinesq approx: $\frac{1}{\rho} \to \frac{1}{\rho_0}$

Coriolis: rotating frame of reference

For Boussinesq flow with $(H/L)^2 \ll 1$

scaling shows $\boxed{z\text{-mom}}$ is ~ "hydrostatic"

$$0 = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{g\rho}{\rho_0}$$

$\boxed{z\text{ mom}}$ $$\boxed{\frac{\partial p}{\partial z} = -\rho g}$$

Another consequence of Boussinesq approx.

is that $\boxed{\text{mass}}$ $\frac{1}{\rho}\frac{D\rho}{Dt} + (\nabla \cdot \underline{u}) = 0$

is approximately <u>incompressible</u>

$\boxed{\text{mass}}$ $\nabla \cdot \underline{u} = 0$

Nevertheless, we have to keep track of the small density variation ρ' because it affects $\frac{\partial p}{\partial x}$ & $\frac{\partial p}{\partial y}$

Simplified equation of state: $\rho(S, T, p) \to \rho(S)$

$\boxed{\rho}$ $$\rho = \rho_0 (1 + \beta S) \quad , \quad \beta = 7.7 \times 10^{-4}$$

where $S = \dfrac{\text{g salt}}{\text{kg seawater}} \sim 0 - 35 \; (\text{g/kg})$

"Absolute salinity"

And S also has turbulent diffusion:

$\boxed{S}$ $$\frac{DS}{Dt} \cong \frac{\partial}{\partial z}\left(K \frac{\partial S}{\partial z}\right)$$

eddy diffusivity $\left(\frac{m^2}{s}\right)$

similar in scale to A

So we have 6 equations:

$\underset{(3)}{\boxed{\underline{x} \text{ mom}}}$ $\boxed{\text{mass}}$ $\boxed{\rho}$ $\boxed{s}$

in 6 unknowns:

u, v, w, p, ρ, s ✓ yay!